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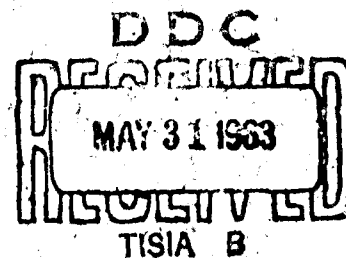
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## **Interim Research Memorandum**

**OPERATIONS EVALUATION GROUP**

WASHINGTON 25, D. C.

DEPARTMENT OF THE NAVY  
OFFICE OF THE CHIEF OF NAVAL OPERATIONS  
WASHINGTON 25, D. C.

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Encl: (1) OEG IRM-36 "Bayesian Supply Policies for Service-Life Parts" Unclassified of 27 Mar 1963

1. Enclosure (1) is forwarded herewith for your information and retention.

2. This Memorandum examines some of the theoretical and practical implications of applying Bayesian techniques to the logistics of service-life parts. The characteristic feature of service-life parts, from the standpoint of inventory theory in particular and of logistics management in general, is that demands (i.e., parts failure) are not generated by a Poisson process, because failures are not independent of age. The applicable statistical models are those of renewal theory and replacement theory. Where uncertainty exists concerning the parameters of the underlying failure distribution, the techniques of Bayesian decision theory may be advantageously applied.

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For the Director  
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27 March 1963

INTERIM RESEARCH MEMORANDUM  
OPERATIONS EVALUATION GROUP

BAYESIAN SUPPLY POLICIES  
FOR SERVICE-LIFE PARTS

by T A. Goldman

IRM-36

This is an interim report of continuing research. It does not necessarily represent the opinion of OEG or the U.S. Navy. It may be modified or withdrawn at any time.

ABSTRACT

The characteristic feature of service-life parts, from the standpoint of inventory theory in particular and of logistics management in general, is that demands (i.e., parts failures) are not generated by a Poisson process, because failures are not independent of age. The applicable statistical models are those of renewal theory and replacement theory. Where uncertainty exists concerning the parameters of the underlying failure distribution, the techniques of Bayesian decision theory may be advantageously applied. This memorandum examines some of the theoretical and practical implications of applying Bayesian techniques to the logistics of service-life parts.

Service-life parts are those parts or components which have a well-defined wear-out characteristic. This definition is equivalent to the more technical statement that the conditional probability of failure (also called the hazard) is an increasing function of part age over some or all of its range, and within the age interval of operational interest. The first part of the latter definition excludes parts with a purely Exponential failure distribution, and the latter part excludes those components with a mean age at failure in excess of the active life of the end item in which they are incorporated. The statistical theory applicable to parts in this category is largely comprised under the heading of renewal theory (references (h), (i), and (q)), and applications have been studied under the heading of replacement theory (references (b), (g), (p), and (x)). It does not appear, however, that much attention has been as yet devoted to the implications for an operating logistics system, particularly as regards the interrelations among subsystems, such as inventory, maintenance, provisioning, overhaul, and so forth. While the following discussion is based on and oriented toward military logistics systems, the analysis and conclusions have applications in many non-military systems as well. The civil aviation industry (references (n) and (y)), is a notable example.

It should be noted in particular that inventory theory has been almost entirely based on the assumption that demands are independent in time (reference (a)), which is necessarily not valid for service-life parts. The widespread use of the Poisson distribution for parts demand in inventory theory is one consequence of the assumption in question. Theoretical analysis of maintenance problems in logistics has been much influenced by reliability theories which assume Exponential failure probability (references (dd) and (ee)). This assumption also is inapplicable to service-life parts.

The Weibull distribution is frequently suggested as the characteristic failure distribution for items which do not fail exponentially (reference (o)). The Weibull distribution belongs to the Gamma family of distributions, otherwise known as Pearson's Type III (the Incomplete Gamma Function). It may be noted, in passing, that the Exponential distribution is a limiting case of the Gamma distribution. In practice, failure distributions for complex parts and components are quite likely to be mixtures. The phenomenon known as "infant mortality" is one consequence of such a mixed distribution for failure probabilities (reference (r)). It should be observed that the presence of an Exponential-type component in a mixed distribution is not inconsistent with the classification of a part in the service-life category, as long as all components of the mixture are not of Exponential type.

For the sake of simplicity, let us assume that we are discussing a part with a pure, unimodal failure distribution having service-life characteristics. The operating system will typically consist of large numbers of a particular piece of equipment containing the part under consideration. Each piece of equipment, which might be a ship, a truck, an aircraft, or perhaps a large piece of machinery, constitutes an end item to be supported by the logistics system. When a part fails, it is replaced by a spare part, which we assume to have identical characteristics.



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In this situation, with the specified assumptions, we obtain the classical renewal equation, as follows: The number of replacements at time  $t$  is represented as  $u(t)$ . The probability that a part of age  $k$  will fail before reaching age  $k + dk$  is  $f(k)dk$  (where  $f(k)$  is the failure distribution). The number of parts of age  $k$  is  $u(t-k)$ , i.e. the number of new parts installed  $k$  time units ago. The expected number of failures at time  $t$  due to parts of age  $k$  is thus  $u(t-k)f(k)dk$  and the aggregate number of

failures at time  $t$  for parts of all ages is given by:  $u(t) = \int_0^t u(t-k)f(k)dk$ . In the

general case, allowance is made for parts which were not of age zero at time zero. The number of failures of such parts at time  $t$  is a series  $b(t)$ , giving:

$$u(t) = b(t) + \int_0^t u(t-k)f(k)dk \text{ (references (h) and (q)).}$$

An excellent presentation of

statistical renewal theory is given by Feller (reference (i)), who takes time as a discrete rather than a continuous variable and thus obtains the alternative form:

$$u(t) = b(t) + \sum_{k=0}^t a_k u(t-k) \text{ in which } a_k \text{ is the probability of failure at age } k.$$

A very important result, for a logistics system, of the statistical analysis which renewal theory provides is that a unimodal failure distribution of the service-life type discussed above will give rise to a series of damped oscillations for the number of replacements  $u(t)$  (references (q) and (x)). It is evident that the series  $u(t)$  represents the demands for spare parts which will be experienced by inventory managers, as well as the generation of reparable parts confronting maintenance managers. If demands fluctuate as indicated, then procedures which call for setting stock levels on the basis of average demands over the past three, six, or twelve months (for example) will not give very satisfactory results. More generally, any conclusion or procedure based on the assumption that demands can be considered as independent observations from a probability distribution which is stationary over time will not be valid. The lack of stationarity over time is, moreover, of a special sort, which warrants the application of special procedures.

A consideration of fundamental importance in the logistics treatment of service-life parts is that the variance of the failure distribution will have a major influence on the amplitude and the rate of damping of the oscillations of the renewal function. In the case of parts with a mandatory removal time, the variance derives entirely from failure occurring at ages less than this maximum age. Otherwise, it is influenced by later failures as well as by earlier ones. Since most service-life parts, especially the more expensive ones, are overhauled and returned to inventory for re-use, the maximum amplitude and the rate of damping of replacement requirements (given by the renewal function) are of vital importance in such decisions as the level of "pipeline inventory" and the scheduling of overhaul activities (references (c), (j), and (z)).

A very important factor in logistics planning is the way in which the end items enter the operating system. The term "phase-in schedule" is used for the schedule which states the number of new end items entering the operating system as a function of time. If we represent this function as  $b(t)$ , then the renewal function given above will represent the number of new parts entering the operating system at time  $t$  as  $u(t)$ , and the number of replacements will be given by:  $u(t) - b(t)$ .

The influence of the phase-in schedule on demand for spare parts is of considerable importance. If the situation in which all end items come into operation at the same time is taken as a basis for comparison, then any more gradual phase-in schedule will tend to smooth the fluctuations in demand, lowering and flattening the peaks and, by the same token, raising the troughs. A convex phase-in schedule will have a stronger smoothing effect than a concave one. Of particular interest is the effect of the phase-in schedule on the demand for replacements before the first peak of the renewal function is reached. By reducing the steepness of the upward slope, it provides opportunity for review of provisioning decisions, as well as other, related decisions, in the light of (gradually) increasing information. Any phase-in schedule whatever will have this effect by comparison with the situation in which all end items come into operation at the same time; the more gradual the phase-in, the stronger the effect will be, of course.

If the failure distribution for each service-life part were known precisely, the logistics problems would not be exceptionally severe. There is considerable literature on replacement theory (references (g), (p), and (x)), and the overhaul problem which arises in this context has been formulated, if not completely solved, in the operations-research literature on the "caterer problem" (references (c), (j), (m), and (u)). The most difficult feature of the problem in actual operation of any logistics system, military or industrial, derives from the presence of uncertainty regarding the parameters of the failure distribution, both mean and variance\* (references (n), (y), and (bb)).

The nature of the problems arising from such uncertainty is well illustrated by military experience with early jet aircraft. Lacking prior experience with jet engines, logistics managers underestimated the mean time to failure and therefore set the mandatory removal time (TBO, or time between overhauls) quite low. This decision led to the procurement of large quantities of spare engines for pipeline inventory. As experience accumulated, the TBO was progressively extended to a much higher level. Repeated revision of the mandatory removal time had the same effect as an increase in the variance of the failure distribution. A number of engines were replaced at an early age, before the TBO was extended, others were replaced according to the new TBO requirement before it too was extended, and so on. Underestimation of the mean of the failure distribution, combined with this artificial increase in the variance, resulted in a far lower peak requirement for spare engines than was anticipated in the original procurement decisions. As a consequence, there was substantial overprocurement of spare engines, at a very high unit cost.

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\* If the distribution is not a two-parameter distribution (and the Weibull distribution is not), this statement applies to the other parameters as well.

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Some procedure is needed to deal with the problems arising in logistics systems as a result of uncertainty about the failure distribution and about the demand for spare parts which the failure distribution generates. As it happens, the methods of Bayesian statistics have been developed for just such problems as these (references (f), (w), (bb), and (c)).

The characteristic feature of Bayesian statistical procedures is that an initial estimate (called the prior estimate) of the probability measures which are relevant to the decision problem is made, but is subject to revision on the basis of statistical observations. The revised estimate which results is called the posterior estimate. The application of Bayesian techniques in logistics decisions concerning service-life parts, in the maintenance sector as well as in the supply sector, will merit consideration whenever there are no historical data, i.e. data derived from previous operational experience, for the part in question. The conclusion is also valid, of course, if such data exist but are clearly inadequate in quantity or quality. The crucial feature is that the form of the failure distribution is not known with certainty.

This is the situation which prevailed when automatic transmissions replaced the manual-shift types in automobiles, or when an alternator replaces the standard generator. It is the situation which confronts the operator of a fleet of trucks (or a fleet of aircraft) who changes to a different make or type of equipment. It is a situation which arises in greater or less degree whenever a manufacturer of any type of machinery or equipment introduces a new model or makes a design change in an old model. In particular, it is a very common situation in military systems, where new equipment is constantly being introduced into operating units to replace older, obsolescent equipment.

There are a number of factors which make the utilization of Bayesian procedures especially suitable in connection with service-life parts:

1. Prior estimates are practically indispensable.

None of the decisions which have to be made with respect to service-life parts can be made without some estimated values for the failure distribution or the renewal function or both. (If both are estimated, presumably the estimates are mutually consistent.) In practice, various procedures have been and are being used, by the armed services in particular, to provide such estimates. Some procedures may have better theoretical justification than others, but the point is that no attempt is made, in current practice, to make relevant logistics decisions without some estimate of the probability measures involved.

This observation disposes of one of the most frequently heard objections to Bayesian procedures, namely that they call for prior estimates not derived from statistical observations and analysis. Since the prior estimates (which might equally well be called "initial estimates") are to be utilized in any case, the question is no longer whether but how to make use of them. Moreover, the criticism of prior estimates as tending to be subjective can scarcely be applied to the estimates customarily used in military provisioning, for example. Every effort is made in practice to derive

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needed estimates from sources and considerations of an objective character. Previous experience with similar parts, prototype experience, design considerations, and engineering analysis are among the sources utilized. The resulting estimates can scarcely be characterized as "subjective" or "personalistic." On the other hand, they are no more than prior estimates. Even in the case of previous experience with similar parts, historical data are not usually collected and subjected to statistical analysis, as would be required for a non-Bayesian estimate. Even if this were in fact done, the fact that two parts are similar but not identical precludes the extrapolation of measures of confidence, etc.

Life testing constitutes the only possible exception to the conclusion that prior estimates must be used because statistical analyses are not available. To the extent that life testing yields values for the probability measures which differ from those obtained in operational experience, the exception is more apparent than real. Life testing is discussed in more detail below.

2. Data suitable for statistical analysis are provided by early operating experience.

The failure distribution and the renewal function are not statistical artifacts, but merely mathematical descriptions of events which occur and are observed in the course of normal system operation. It might not be necessary to keep records of the events so observed, but in fact such records usually are kept for other purposes than statistical analysis. The additional costs of collecting and processing the data for purposes of improved logistics decision-making are likely to be no more than nominal. They will certainly be small in proportion to the savings which may result.

Having established that prior estimates are made and are used, and that observations on the relevant probability measures are generated and (usually) recorded, we have reduced the question of using Bayesian techniques to the rather simple level of asking: Should the original prior estimates be revised by means of the observations, or should they be allowed to stand unrevised as a basis for logistics decisions? In the final analysis, the answer to this question should be based on consideration of the costs and the benefits to be anticipated. As pointed out below, one of the principal features and a major advantage of Bayesian statistics is that it explicitly provides for such comparison of costs and benefits in terms of the value of additional information (references (w) and (cc)).

3. Early demand for service-life parts is relatively low (by hypothesis).

The first two points discussed above are as applicable, in general, to parts which are not in the service-life category as to those which are. There are several features which are specific to the case of service-life parts which make the application of Bayesian techniques particularly appropriate. One feature of considerable significance is the relatively low level of demand for service-life parts during the early operational life of the equipment containing them. The resulting benefits are two-fold: On the one hand, a relatively small investment in inventory will give a high level of protection against stock-outs. On the other hand, a relatively high

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(not absolutely high, but high relative to demand) level of inventory is not likely to result in serious excesses in the long run. As a consequence, the cost of additional information with regard to the demand probabilities is low at a time when the value of additional information is high. This situation strongly favors a Bayesian approach to the logistics decision problem.

If the phase-in schedule for the new equipment is gradual, the tendency toward low demand levels in the early time periods will be reinforced. The benefits of a gradual phase-in schedule in this respect will, of course, accrue as well to parts which are not of service-life type.

4. Under Bayesian procedures, an initial period in which no demands at all are observed will nevertheless result in improved estimates of the true demand probabilities for decision-making purposes.

If the true failure distribution is characterized by a long average life (i.e. mean time to failure), or a very small variance about the mean, then the probability that no failures at all will occur at early ages, hence during the initial period of the operating life of the end items, will be relatively high. In Bayesian estimation, the converse of this proposition can be used to revise the initial estimate in the direction of higher mean life, smaller variance by reducing the weight assigned to short mean life and large variance. Hence the information content of observations of zero demand is non-zero. This is a valuable fail-safe aspect of Bayesian procedures.

The procedures of classical statistics simply do not provide for this situation. No failures are necessarily equated to no observations, and therefore no estimate of the failure distribution is possible. The best the classical statistician could do would be to test the hypothesis that the failure distribution had some specified mean and variance, but it would require a very liberal approach to classical statistics even to set up such an hypothesis under those circumstances. Lacking such a liberal viewpoint, the classical statistician might try to compute confidence limits, at least a lower bound, for the survival probability within the time interval covered by operational experience. Since this probability is a binomial one, the estimation procedure is distribution-free. Estimation of such survival probabilities is characteristic of the actuarial approach. Unfortunately, the contribution to logistics decisions is minimal as long as no failures are observed.

In the case where true infant mortality (as defined earlier) is a significant element of total failures, and consequently failures are not zero during the early operational history of the equipment, there is something to be gained from application of non-Bayesian statistical procedures. Even in this case, there is still more to be gained from the methods of Bayesian decision theory, however.\*

\* The above discussion should serve incidentally to clarify the difference in viewpoint between the present article and that of Radner and McGlothlin (reference (v)). In discussing service-life parts, they present a modified version of the actuarial approach as described above. They correctly refer to their estimating procedure as Bayesian, because it is based on an initial or prior estimate which is to be revised by means of data derived from operational experience. The present article is concerned with Bayesian decision theory, as distinguished from Bayesian estimation. The principal difference is that the decision-theoretical approach calls for explicit consideration of the cost function, while the estimating procedure does not.

5. Bayesian procedures make it possible to assess the net value of increasing information.

At some point, it ceases to be worthwhile to defer decisions in order to obtain improved estimates of failure probabilities. The procedures of Bayesian decision theory provide for an explicit evaluation of the expected costs and benefits of increasing information concerning the probability measures (reference (cc)). As an important special case, there may be some service-life parts for which the costs of obtaining additional information concerning failure probabilities will prove to be greater than the savings to be anticipated. This is most likely to be true of parts with relatively low unit cost.

Another problem of particular interest is that of life-testing. Service-life parts are often--perhaps typically--large, complex, mechanical assemblies of high unit cost. For such parts, life-testing is likely to be relatively expensive as compared to the case of small electronic components. The benefits to be derived from life-testing come from lower system costs for supply and maintenance as a result of reductions in uncertainty about the failure distribution. To assess the net value of increasing information from life-testing procedures, benefits must be weighed against costs. The statistical reliability of life-test data, and therefore the reduction in uncertainty, will usually be a function of sample size. Costs will also be a function of sample size, in most cases, so that there will be a direct trade-off between costs and benefits. Bayesian decision theory, by making this trade-off explicit in the form of a cost function, permits an optimization of the decision as to whether to test, and if so, what sample size to select (references (w) and (cc)).

6. For service-life parts, Bayesian estimation provides the only satisfactory forecasting procedure.

For parts with demand probabilities of the Poisson type, or at least a probability distribution which is stationary over time\*, the methods of classical statistics can be used to obtain forecasts of future demand from early operational experience (references (e), (k), and (l)). Where service-life parts are concerned, this is not the case. Classical procedures would not provide an estimate of the failure distribution suitable for forecasting purposes until a large number of parts had failed. An estimate of the median service life, for example, could not be made until at least half of a reasonably large sample of parts had failed. An estimate of the mean service life, from the same sample, would take even longer.

Methods have been developed, within the framework of classical statistics, for estimating the parameters of a probability distribution from truncated samples (references (t) and (aa)), but these require that the distribution be of known type. For the problem here considered, this is a Bayesian assumption in itself. The actuarial approach, of estimating the failure probability for each age interval separately, requires no assumption about the distribution (i. e. is distribution-free), but takes much longer, just for that reason, to produce an adequate forecast.

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\* This category includes those parts for which demand probabilities are a stationary function of the total number of end items per time period, or, as in the case of certain electronic parts, of the total number of operating hours per time period, even though the number of end items or operating hours are not constant over time.

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It should again be emphasized that estimation and forecasting techniques, while of interest and indeed of importance, are here regarded as secondary to the decision-making procedure which constitutes the principal contribution of modern Bayesian statistics. From the initial estimate to the final cut-off point, the Bayesian analysis enables the decision-maker to find the best possible decision on the basis of the information available to him at the time.

What specific policies and procedures are called for in order to implement a Bayesian approach to the logistics of service-life parts? The answer to this question can be summarized rather simply. The following summary is in terms of a military logistics system, but the application to an industrial system is straightforward. A basic difference arises from the division of logistics functions between the user of the equipment and the producer of the equipment, in most non-military situations. In the civil aviation field, this has led to various forms of organized cooperation between the individual aircraft manufacturing firm and its airline customers.

1. The fundamental requirement, not only of Bayesian techniques but of any really sensible approach, is explicit recognition of service-life parts as a separate category requiring appropriate procedures. Setting inventory levels and procurement quantities on the basis of mean time between overhauls, for example, is clearly non-optimal for service-life parts in general, because it fails to allow for peaks (and troughs) in demand.

Optimal policies for service-life parts are characterized in particular by special treatment of peak periods in the demand cycle. Peak inventory requirements may be met by use of expedited repair as an alternative to additional procurement (reference (ee)). Peak maintenance (M & O) requirements may be met by more intensive utilization of available overhaul capacity as an alternative to investment in additional capacity. This may involve increased overtime work, substitution of continuous-flow processing for batching, temporary reallocation of personnel and equipment, priority scheduling of peakload items, and other forms of intensification.

Special consideration may need to be given to trade-offs between inventory costs and maintenance costs. The peak load on maintenance capacity can be somewhat smoothed by instituting expedited repair procedures before inventory considerations (taken by themselves) would require it. Temporarily higher inventory levels will result, so that storage costs are substituted for the costs of intensive overhaul activity. Where mandatory removal of a part is involved, systematic extension of the mandatory removal time at the first peak, when this measure can be justified on the basis of experience to date and other relevant considerations, will serve to flatten out the peak demand to some degree (reference (n)).

While peak demand levels place substantial strains on the system, troughs in the demand cycle present only the danger of unwarranted extrapolation of low demand levels as a forecast of future demands. The danger is a real one in

military logistics systems, where operating procedures are based on a highly specific set of regulations. Whenever regulations and procedures assume, implicitly or explicitly, a stationary probability distribution of parts demand, they may lead to serious difficulties in the management of service-life parts.

2. Early logistics decisions should be made subject to later revision as needed.

Unless decisions taken early in the life-history of a new system can be reviewed and revised in the light of subsequent information, the decision maker has no freedom of action. Even without the application of strictly Bayesian techniques, there are substantial advantages in retaining such freedom of action. Such advantages provide the justification of proposals for deferred procurement, for example, which offer the prospect of substantial savings in themselves (reference (s)). Such savings will be increased by improvements in the decision-making procedure like those suggested here. Those familiar with current statistical concepts are aware that Bayesian estimation and Bayesian decision theory can be extended to include the situation in which a decision is made on the basis of the prior estimate alone, without the possibility of subsequent revision in the light of posterior estimates. Indeed, the ability of Bayesian decision theory to cover this type of situation is often regarded as one of its great advantages (references (f) and (cc)). As indicated earlier, this type of "Bayesian" decision making, albeit in a rather crude form, is what the armed services have had to engage in all along. They are in the position of Moliere's M. Proudhon, who had been speaking prose all his life without knowing it. A formalized decision-making procedure, with the possibility of revised decisions based on posterior estimates, can thus represent a substantial step forward.

A system which provides for flexibility and quick adjustment in early decision making puts substantive content into the word "calculated" in the concept of calculated risk. As pointed out above, it will usually be advantageous to maintain a relatively high level of inventory support for service-life parts during the early operational period, but the costs of priority action can be explicitly calculated in determining optimum inventory levels, when Bayesian decision-making procedures are employed. A small risk of expedited procurement or expedited repair can be traded off against a potentially greater risk of over-procurement, for example.

3. Early demand experience should be collected and analyzed, to the extent warranted by the attainable savings in system cost.

Statistical analysis necessarily requires data as its basic input material. In Bayesian statistics, such data are necessary in order to move from prior estimates to posterior estimates. As indicated above, it is the development of posterior estimates from early experience which constitutes the justification for revision of initial logistics decisions. When initial decisions have been subject to revision in



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current practice, the revised policies have usually been based on informal analysis of early experience, often no more than a general impression of what was happening. In these circumstances, the revised decision is not likely to be optimal, and may sometimes have merely resulted in piling one costly error on top of a previous one. The alternation of peaks and troughs in demand for service-life parts can easily lead to errors of over-compensation. The classic example is the oft-quoted but probably apocryphal case of a part which, after a protracted initial period in which inventory levels were considerably in excess of actual demand, was disposed of as surplus in accordance with standard procedures, just before a heavy surge of demand occurred, which therefore required additional procurement. Mechanical application of the inventory procedures which would produce such a situation might even lead to repeated cycles of disposal and procurement. One may doubt that human beings would fall into this trap, but it is such possibilities that make special safeguards necessary when electronic computers are involved.

If the statistical analysis is aimed at producing an estimate of the failure distribution, the data collected must include the age of the part at failure and appropriate identifying information. If the objective is an estimate of the renewal function, it is merely necessary to know the time when the failure occurred and the phase-in schedule for the end items (reference (d)). The latter approach simplifies the task of data collection in most logistics systems, but it requires considerably longer to achieve any given level of statistical reliability. In addition, the statistical mathematics for the estimation problem gets more complicated in the latter case.

4. Determine the cost elements of the particular operating system, and evaluate explicitly the costs and benefits associated with each decision.

There has been so much discussion of cost functions and of cost analysis in the literature of operations research in general and of inventory theory in particular that detailed discussion of the subject here would be a waste of time and ink. It should be emphasized, nevertheless, that a cost function (of some kind) is an essential element of Bayesian decision theory (and indeed of decision theory in general). None of the theoretical constructs introduced in the earlier sections of this memorandum can be implemented without some cost information.

It is very instructive for someone familiar with the modern literature on Bayesian decision theory to observe logistics decision-making in practice in such a manifestation as a military provisioning conference. Military regulations and procedure manuals tend to reduce the elements of uncertainty to a minimum, usually by reliance on averages (over time), and to treat the decision problem as though the inputs were determinate. In the decision-making process, however, responsible individuals give clear evidence of their awareness of random variability and uncertainty, especially in a conference situation. Subjective probabilities are much in evidence, but so also are subjective cost functions. It is the contention of this memorandum that better decisions can be made if both probabilities and costs are stated as explicitly, and hence as accurately, as possible, and if costs and probabilities are systematically and scientifically integrated with each other in the decision-making process.

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